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Semi-infinite Throat as the End-state Geometry of two-dimensional Black Hole Evaporation

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ABSTRACT: We study a modified two-dimensional dilaton gravity theory which is exactly solvable in the semiclassical approximation including back-reaction. The vacuum solutions of this modified theory are asymptotically flat static space-times. Infalling matter forms a black hole if its energy is above a certain threshold. The black hole singularity is initially hidden behind a timelike apparent horizon. As the black hole evaporates by emitting Hawking radiation, the singularity meets the shrinking horizon in finite retarded time to become naked. A natural boundary condition exists at the naked singularity such that for general infalling matter-configurations the evaporating black hole geometries can be matched continuously to a unique static end-state geometry. This end-state geometry is asymptotically flat at its right spatial infinity, while its left spatial infinity is a semi-infinite throat extending into the strong coupling region.

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Hawking's discovery that black holes radiate thermally [[1],[2],[3]] gave rise to a long-standing question concerning the consequences of combining quantum theory and general relativity. [[4],[5],[6],[7]] Does evolution from an initial pure state take place unitarily to a final pure state or non-unitarily to a final mixed state? Intimately linked to this question is the final geometry resulting from black hole evaporation.

Here we present a specific two-dimensional (2D) dilaton gravity model in which a black hole evaporates leaving a static semi-infinite throat as the end-state or "remnant" geometry. Our model is a modification of the CGHS model.[[8]] We solve the semiclassical equations and get closed-form expressions for the metric and dilaton field.

The classical 2D CGHS action [8] is

$$S_{c\ell} = \frac{1}{2\pi}\int d^2x\sqrt{-g}\left[e^{-2\phi}\left(R^{(2)} + 4(\nabla\phi)^2 + 4\lambda^2\right) - \frac{1}{2}\sum_{i=1}^{N}(\nabla f_i)^2\right], \tag{1}$$

where ϕ is the dilaton field, $R^{(2)}$ is the 2D Ricci scalar, λ is a positive constant, ∇ is the covariant derivative, and the f_i are N matter (massless scalar) fields. The action (1) describes a 2D effective theory in the throat region of a 4D almost extreme magnetically charged black hole.[[9],[10]] It may also be regarded as a 2D arena in which some of the main questions about black hole evaporation can be studied. Among the classical solutions stemming from the action (1) are vacuum solutions, static black hole solutions, and dynamical solutions describing the formation of a black hole by collapsing matter fields. For a review see Ref. [[11]].

To study one loop quantum corrections and back-reaction one can use the trace-anomaly for massless scalar fields in two dimensions, $\langle T^{\mu}_{\mu}\rangle = \frac{\hbar}{24}R^{(2)}$, and find the effective action S_{PL} for which $\langle T_{\mu\nu}\rangle = -\frac{2\pi}{\sqrt{-g}}\frac{\delta}{\delta g^{\mu\nu}}S_{PL}$. This is the Polyakov-Liouville action [[12]]

$$S_{PL} = -\frac{\hbar}{96\pi}\int d^2x\sqrt{-g(x)}\int d^2x'\sqrt{-g(x')}R^{(2)}(x)G(x,x')R^{(2)}(x'), \tag{2}$$

where $G(x,x')$ is a Green function for ∇^2. Here we take the large N limit, in which $\hbar$ goes to zero while $N\hbar$ is held fixed. In that limit the quantum corrections for the gravitational and dilaton fields are negligible, and one need take into account only the quantum corrections for the matter (scalar) fields. The one-loop effective action is then $S_{(1)} = S_{c\ell} + NS_{PL}$. There are no known analytic solutions to this one-loop effective theory, though there are some numerical ones.[[13]] In order to find analytic solutions including semiclassical corrections, one can modify the action as in [[14],[15],[16]]. Our approach

is similar, in that we modify the original CGHS action (1) and find analytic solutions to the modified equations including back-reaction. However, our analytic solutions yield *closed-form* expressions for the metric and dilaton field. This allows us to fully analyze the solutions.

We add to the classical action (1) a local covariant term of one-loop order,

$$S_{\text{corr}} = \frac{N\hbar}{24\pi} \int d^2x \sqrt{-g} \left((\nabla\phi)^2 - \phi R^{(2)} \right). \tag{3}$$

Now the total modified action including the one-loop Polyakov-Liouville term is

$$S_{\text{mod}} = S_{c\ell} + S_{\text{corr}} + N S_{PL}. \tag{4}$$

Using null coordinates $z^{\pm}$ and conformal gauge $g_{++} = g_{--} = 0$, $g_{+-} = -\frac{1}{2}e^{2\rho}$ ($ds^2 = -e^{2\rho}dz^+dz^-$), the action (4) can be written in the form

$$S_{\text{mod}} = \frac{1}{\pi} \int dz^+dz^- \left[2\partial_-(\phi-\rho)\partial_+ \left(e^{-2\phi} - \frac{\kappa}{2}(\phi-\rho) \right) + \lambda^2 e^{2(\rho-\phi)} + \frac{1}{2}\sum_{i=1}^{N} \partial_+ f_i \partial_- f_i \right], \tag{5}$$

where $\kappa = \frac{N\hbar}{12}$. [From the point of view of string theory, the action (5) with free fields $X \equiv e^{-2\phi}$ and $Y \equiv \phi - \rho$ (which are flat target space coordinates), describes a conformal field theory with tachyon and dilaton backgrounds $\mathrm{T} = -4\lambda^2 e^{-2Y}$ and $\Phi = -2X + 2\kappa Y$.[[17]]] The action (5) is also invariant under the transformation† [14] $\delta\phi = \delta\rho = \epsilon e^{2\phi}$, with the conservation equation $\partial_\mu\partial^\mu(\phi - \rho) = 0$. We therefore can complete the gauge fixing by choosing the "Kruskal coordinates," $x^{\pm}(z^{\pm})$, in which $\phi(x^+, x^-) = \rho(x^+, x^-)$. In this Kruskal gauge the equations of motion derived from the modified action (5) are *exactly* the same as the classical ones

$$\partial_{x^+}\partial_{x^-} \left(e^{-2\rho(x^+,x^-)} \right) = \partial_{x^+}\partial_{x^-} \left(e^{-2\phi(x^+,x^-)} \right) = -\lambda^2 \tag{6}$$

$$\partial_{x^+}\partial_{x^-} f_i(x^+, x^-) = 0, \tag{7}$$

while the constraints get modified by non-local terms $t_{\pm}(x^{\pm})$ arising from the Polyakov-Liouville action. In conformal gauge, one can use the trace anomaly of N massless scalar

† Unlike in the RST model, [14], in this model the transformation is *exactly* the same as in the classical case.

fields f_i to obtain $\langle T^f_{+-}\rangle = -\kappa\partial_+\partial_-\rho$ and integrate [[18],[19],[20]] the equation $\nabla^\mu \langle T^f_{\mu\nu}\rangle = 0$ to get the quantum corrections to the energy-momentum tensor of the f_i matter fields

$$\langle T^f_{\pm\pm}\rangle = \kappa\left(\partial^2_\pm\rho - (\partial_\pm\rho)^2 - t_\pm(z^\pm)\right), \tag{8}$$

where $t_\pm(z^\pm)$ are integration functions determined by the specific quantum state $|\Psi\rangle$ corresponding to the expectation value $\langle\Psi|T^f_{\mu\nu}|\Psi\rangle \equiv \langle T^f_{\mu\nu}\rangle$. These functions can be determined by boundary conditions. Alternatively, Eq. (8) can be obtained by varying NS_{PL}. Then the functions $t_\pm(z^\pm)$ arise from the homogeneous part of the Green function in Eq. (2). Our modified constraints (in Kruskal gauge) are

$$\frac{\delta S_{\text{mod}}}{\delta g^{\pm\pm}} = 0 \;\Rightarrow\; -\partial^2_{x^\pm}\left(e^{-2\phi(x^+,x^-)}\right) - (T^f_{\pm\pm})_{c\ell} + \kappa t_\pm(x^\pm) = 0, \tag{9}$$

where $(T^f_{\pm\pm})_{c\ell} = \frac{1}{2}\sum_{i=1}^N \left(\partial_{x^\pm} f_i\right)^2$ is the classical (zero order in $\hbar$) contribution to the energy-momentum tensor of the f_i matter fields. $\langle T^f_{\mu\nu}\rangle$ in (8) is the one-loop quantum correction of order $\hbar$, so the full energy-momentum tensor of the f-fields is $(T^f_{\mu\nu})_{c\ell} + \langle T^f_{\mu\nu}\rangle + O(\hbar^2)$.

For a given classical matter distribution and a given $t_\pm(x^\pm)$ one finds the solution for the equations of motion (6) with the constraints (9):

$$\begin{aligned} e^{-2\phi} = e^{-2\rho} = -\lambda^2 x^+ x^- &- \int^{x^+} dx_2^+ \int^{x_2^+} dx_1^+ \left[(T^f_{++})_{c\ell} - \kappa t_+(x_1^+)\right] \\ &- \int^{x^-} dx_2^- \int^{x_2^-} dx_1^- \left[(T^f_{--})_{c\ell} - \kappa t_-(x_1^-)\right] + a_+ x^+ + a_- x^- + b \end{aligned} \tag{10}$$

where $a_\pm$ and b are constants. First, let us consider the linear dilaton flat space-time solution, $e^{-2\phi} = e^{-2\rho} = -\lambda^2 x^+ x^-$. It corresponds to the choice $(T^f_{\mu\nu})_{c\ell} = 0$ and $t_\pm(x^\pm) = a_\pm = b = 0$. To determine the corresponding quantum state $|\Psi\rangle$ one must calculate $\langle T^f_{\pm\pm}\rangle$ in (8) using the given $t_\pm(x^\pm)$. In flat coordinates $\sigma^\pm$, which are related to the Kruskal coordinates $x^\pm$ by the conformal coordinate transformation $\pm\lambda x^\pm = e^{\pm\lambda\sigma^\pm}$, the expectation values (8) are $\langle T^f_{\pm\pm}(\sigma^\pm)\rangle = \frac{\kappa\lambda^2}{4}$. We see that unlike in the RST model, in our model $\langle T^f_{\pm\pm}(\sigma^\pm)\rangle \neq 0$ for the linear dilaton solution. Because $\langle T^f_{\pm\pm}\rangle = \frac{\kappa\lambda^2}{4}$ and $\langle T^f_{+-}\rangle = 0$, the quantum state $|\Psi\rangle$ corresponding to the linear dilaton solution may describe a system in thermal equilibrium at temperature $T = \frac{\lambda}{2\pi}$.

In our model we also have *static* black hole solutions.[[21]] These correspond in Eq. (10) to the choice $(T^f_{\mu\nu})_{c\ell} = t_\pm(x^\pm) = a_\pm = 0$ and $b = M/\lambda$. For these solutions at future and

past null infinity ($\Im^+$ and $\Im^-$, respectively) one has $\langle T^f_{\pm\pm}\rangle = \frac{\kappa\lambda^2}{4}$; the solutions evidently describe a black hole in thermal equilibrium at temperature[‡] $T_{bh} = \frac{\lambda}{2\pi}$. This is as we would expect: A static black hole solution in a self-consistent semiclassical theory of Hawking radiation including back-reaction is possible only if the black hole is in thermal equilibrium with incoming radiation.

In order to find the solution corresponding asymptotically to the Minkowski vacuum we can use (8) to find the solution for which $\langle T^f_{\pm\pm}(\sigma^\pm)\rangle = 0$. The functions $t_\pm(x^\pm)$ are determined by imposing appropriate boundary conditions on $\Im^\pm$. We assume that on these boundaries the metric is flat, such that $\rho(\sigma^\pm)$ and its derivatives vanish in the asymptotically flat coordinates $\sigma^\pm$. Then the first two terms on the right-hand-side of (8) vanish on the boundary and we get

$$\langle T^f_{\pm\pm}(\sigma^\pm)\rangle|_{\text{boundary}} = -\kappa t_\pm(\sigma^\pm), \tag{11}$$

We see from (11) that the Minkowski vacuum corresponds to $t_\pm(\sigma^\pm) = 0$. To find the corresponding $t_\pm(x^\pm)$ in "Kruskal coordinates," one can use the tensor transformation of $\langle T^f_{\pm\pm}\rangle$ in Eq. (8) (under a conformal coordinate transformation) and get

$$t_\pm(x^\pm) = \left(\frac{\partial\sigma^\pm}{\partial x^\pm}\right)^2 \left(t_\pm(\sigma^\pm) - \frac{1}{2}D^S_{\sigma^\pm}[x^\pm]\right) = \frac{1}{(2x^\pm)^2}, \tag{12}$$

where $D^S_y[z]$ is the Schwarz operator $D^S_y[z] = \partial^3_y z/(\partial_y z) - \frac{3}{2}\left(\partial^2_y z/\partial_y z\right)^2$ and we use $t_\pm(\sigma^\pm) = 0$. Using (10), (12) and $(T^f_{\mu\nu})_{c\ell} = 0$, we find that the general asymptotically Minkowski vacuum solution is

$$e^{-2\phi} = e^{-2\rho} = -\lambda^2 x^+ x^- - \frac{\kappa}{4}\log(-\lambda^2 x^+ x^-) + C, \tag{13}$$

where C is a constant. In asymptotically flat coordinates $\sigma^\pm = t \pm \sigma$, we have

$$\begin{aligned} ds^2 &= \left(1 - e^{-2\lambda\sigma}(\kappa\lambda\sigma/2 - C)\right)^{-1}(-dt^2 + d\sigma^2) \\ \phi(\sigma) &= -\lambda\sigma + \log\left(1 - e^{-2\lambda\sigma}(\frac{\kappa\lambda}{2}\sigma - C)\right). \end{aligned} \tag{14}$$

‡ Since in 2D the Hawking temperature is mass independent, one may regard the linear dilaton solution as the zero mass limit of the static black hole solutions. This may explain the non-zero temperature of the linear dilaton solution in our model.

This solution is static, depending on the spatial coordinate σ alone. On the boundaries $\Im^{\pm}$, the solution approaches the linear dilaton flat space-time solution, justifying our assumption. The reason this solution with no radiation at $\Im^{\pm}$ and the earlier ones with radiation there all asymptotically approach the linear dilaton flat space-time solution is that the coupling, $e^{2\phi}$, of the matter to the geometry vanishes exponentially fast at $\Im^{\pm}$.

Before we turn to the question of the ground-state solution, let us consider the ADM masses of the various solutions we have found. Suppose that we can choose as our ground-state one of the radiationless solutions (14) with $C = C_0$, where C_0 is a constant yet to be determined. Then the ADM mass [[22],[23]] of any other static solution (14) is $\lambda(C - C_0)$. On the other hand, the ADM mass of the linear dilaton solution as well as the static black hole solutions (relative to this ground-state) is infinite. This is already clear from the fact that these solutions have non-vanishing radiation on $\Im^{\pm}$ and can be checked explicitly by using the ADM mass definition.[22] These considerations make it plausible that one of the static solutions (14) should be the ground-state. We will see later that there exists a natural lower limit on C_0 which gives the preferred ground-state of lowest ADM mass.

We next turn to the dynamical scenario in which the space-time is initially described by one of the static solutions in (14) (not necessarily the ground state solution C_0), and in which a black hole is formed by collapsing matter fields. First we consider the simple shock wave solution, but our results can be easily extended to general infalling matter configurations. The shock wave of infalling matter is described by $(T^f_{++})_{c\ell} = \frac{M}{\lambda x_0^+}\delta(x^+ - x_0^+)$ and $(T^f_{--})_{c\ell} = 0$.[8] Unlike in the RST model, here we have a general initial static geometry, and the shock wave forms a black hole only if M, the energy of the shock wave, is above a certain threshold energy. We assume that M is above that threshold. Integrating $(T^f_{++})_{c\ell}$ in (10) and using (12) and $a_\pm = 0$, we find the evaporating black hole solution

$$e^{-2\phi} = e^{-2\rho} = -\lambda^2 x^+ x^- - \frac{\kappa}{4}\log(-\lambda^2 x^+ x^-) - \frac{M}{\lambda x_0^+}(x^+ - x_0^+)\Theta(x^+ - x_0^+) + C, \qquad (15)$$

where $\Theta(x)$ is the standard step function.

Before the shock wave, i.e., in the region $x^+ < x_0^+$, we have a static solution (14) which is not globally flat. If $\frac{\kappa}{4}[1 - \log(\kappa/4)] + C < 0$, then the scalar curvature *diverges* on a timelike curve $\sigma = \sigma_s$, for which $e^{-2\phi(\sigma_s)} = 0$. Of course this is a region of strong coupling, and one would expect to have higher order quantum corrections there. On the other hand, if $\frac{\kappa}{4}[1-\log(\kappa/4)]+C > 0$, the scalar curvature is bounded. Then the region $\{x^+ \geq 0, x^- \leq 0\}$ is geodesically incomplete and one can analytically extend it to $x^- > 0$ and $x^+ < 0$. Also

in this case there is a region of strong coupling near $\sigma = \sigma_{min} = -\frac{1}{2\lambda}\log(\frac{\kappa}{4})$. In the semiclassical approximation, one avoids the strong coupling region by imposing boundary conditions on a suitable time-like hypersurface.[[24],14,[25]] For the static solutions (14), $\langle T^f_{++}(\sigma^\pm)\rangle$ and $\langle T^f_{--}(\sigma^\pm)\rangle$ are constant on *any* time-like hypersurface $\sigma =$ const. Moreover

$$\langle T^f_{++}(\sigma^\pm)\rangle\big|_{\sigma=\sigma_0} = \langle T^f_{--}(\sigma^\pm)\rangle\big|_{\sigma=\sigma_0} \qquad \text{for any constant } \sigma_0. \tag{16}$$

This means that we can limit our model to a region in which the semiclassical approximation is valid by imposing reflecting boundary conditions (16) on any time-like hypersurface $\sigma = \sigma_0$ that lies outside the region of strong coupling (these boundary conditions are also conformal [25]). The geometry before the shock wave is therefore a static geometry, defined in the region $\sigma > \sigma_s$ in the case $\frac{\kappa}{4}[1 - \log(\kappa/4)] + C < 0$ (or $\sigma > \sigma_{min}$ in the case $\frac{\kappa}{4}[1 - \log(\kappa/4)] + C > 0$), with reflecting boundary conditions on $\sigma = \sigma_s + \delta$ (or on $\sigma = \sigma_{min} + \delta$) where δ is an arbitrary small positive constant.

The solution to the future of the shock wave ($x^+ > x_0^+$) is (see (15))

$$e^{-2\phi} = e^{-2\rho} = -\lambda^2 x^+(x^- + \Delta) - \frac{\kappa}{4}\log(-\lambda^2 x^+ x^-) + \frac{M}{\lambda} + C, \tag{17}$$

where $\Delta = \frac{M}{\lambda^3 x_0^+}$. This solution is asymptotically flat and describes a black hole with a singularity at $e^{-2\phi} = 0$. The black hole singularity curve is

$$-\lambda^2 x_s^+(x_s^- + \Delta) - \frac{\kappa}{4}\log(-\lambda^2 x_s^+ x_s^-) + \frac{M}{\lambda} + C = 0. \tag{18}$$

Initially the singularity is behind an apparent horizon $\partial_+ e^{-2\phi} = 0$,[[26],[10]] which is the curve

$$-\lambda^2 x_h^+(x_h^- + \Delta) = \frac{\kappa}{4}. \tag{19}$$

When the apparent horizon is formed, the black hole starts radiating. One can see this by calculating $\langle T^f_{\mu\nu}\rangle$ at future null infinity ($x^+ \to \infty$). From (17) we see that the asymptotically flat coordinates on $\Im^+$ are $\widehat{\sigma}^\pm$, related to $x^\pm$ by the conformal coordinate transformation, $\lambda\widehat{\sigma}^+ = \log(\lambda x^+)$ and $-\lambda\widehat{\sigma}^- = \log(-\lambda(x^- + \Delta))$. Using (11) and (12) we get

$$\langle T^f_{--}(\widehat{\sigma}^\pm)\rangle\big|_{\Im^+} = \frac{\kappa\lambda^2}{4}\left(1 - \frac{1}{(1 + \lambda\Delta e^{\lambda\hat{\sigma}^-})^2}\right). \tag{20}$$

This is the "standard" Hawking radiation in 2D, where the Hawking temperature $T_H = \frac{\lambda}{2\pi}$ is a constant.[8] One can further verify that when the black hole evaporates over a

long period of time, i.e., if $M >> \kappa\lambda$, the spectrum of the Hawking radiation is indeed Planckian.[[2],[27]]

As the black hole evaporates by emitting Hawking radiation, the apparent horizon shrinks and eventually meets the singularity in a *finite* proper time. They intersect at (see Fig. 1)

$$x^+_{int} = \frac{1}{\lambda^2\Delta}\left(e^{(\frac{4(M+\lambda C)}{\kappa\lambda}+1)} - \frac{\kappa}{4}\right) \qquad \text{and} \qquad x^-_{int} = -\Delta\Big(1 - \frac{\kappa}{4}e^{-(\frac{4(M+\lambda C)}{\kappa\lambda}+1)}\Big)^{-1}. \quad (21)$$

At this point the singularity becomes naked. We show that it is possible to impose a boundary condition in which a weak shock wave emanates from the intersection point, resulting in a solution that is stable (having non-negative ADM mass), conserves energy, and is continuous with the metric defined to the past of the null hypersurface $x^- = x^-_{int}$.

Before considering the solution to the future of the null hypersurface $x^- = x^-_{int}$ (the end-state solution), we calculate the total amount E_{rad} of energy radiated during the evaporation. Integrating (20) over $\Im^+$ (up to x^-_{int}) gives

$$E_{rad} = \int_{-\infty}^{\hat{\sigma}^-_{int}} \langle T^f_{--}(\widehat{\sigma}^-)\rangle d\widehat{\sigma}^- = M + \lambda C - \frac{\kappa\lambda}{4}\left(\log(\kappa/4) - 1\right) - \frac{\kappa\lambda\Delta}{4x^-_{int}}, \quad (22)$$

where $\widehat{\sigma}^-_{int} = \widehat{\sigma}^-(x^-_{int})$. The result (22) is *exact*. The ADM mass [22] of the dynamical solution (15) (relative to the ground state $C = C_0$) is $M_{ADM} = M + \lambda(C - C_0)$. We see that the black hole radiates almost all of its initial energy. The unradiated mass δM remaining as $x^- \to x^-_{int}$ (which is the Bondi mass) is

$$\delta M = M_{ADM} - E_{rad} = \frac{\kappa\lambda}{4}\left(\log(\kappa/4) - 1\right) - \lambda C_0 + \frac{\kappa\lambda\Delta}{4x^-_{int}}. \quad (23)$$

We now consider the solution to the future of the point of intersection (x^+_{int}, x^-_{int}). A natural candidate for such an end-state in our model is one of the static solutions (14), so we try to find boundary conditions such that the solution (17) is continuously matched to one of the static solutions (14). Remember that the asymptotically flat coordinates are $\widehat{\sigma}^\pm$, so one should replace σ in (14) with $\widehat{\sigma} = \frac{1}{2}(\widehat{\sigma}^+ - \widehat{\sigma}^-)$. In the $x^\pm$ coordinates the corresponding static solution is (see (13))

$$e^{-2\phi} = e^{-2\rho} = -\lambda^2 x^+(x^- + \Delta) - \frac{\kappa}{4}\log(-\lambda^2 x^+(x^- + \Delta)) + \widehat{C}. \quad (24)$$

We would like to see if there exists a constant $\widehat{C} = C^*$, such that on the null hypersurface $x^- = x^-_{int}$ the solutions (17) and (24) can be matched continuously. This is indeed the

case and from (21), (17) and (24) we get $C^* = -\frac{\kappa}{4}(1 - \log(\kappa/4))$. The end-state solution, or "remnant", is therefore

$$e^{-2\phi} = e^{-2\rho} = -\lambda^2 x^+(x^- + \Delta) - \frac{\kappa}{4}\log(-\lambda^2 x^+(x^- + \Delta)) - \frac{\kappa}{4}(1 - \log(\kappa/4)), \tag{25}$$

where $x^- > x^-_{int}$. From the constraint equations (9) we find that

$$(T^f_{--}(\widehat{\sigma}^-))_{c\ell} = \frac{1}{2}\sum_{i=1}^{N}(\partial_- f_i)^2 = \frac{\kappa\lambda\Delta}{4x^-_{int}}\delta(\widehat{\sigma}^- - \widehat{\sigma}^-_{int}). \tag{26}$$

This describes a shock wave originating at the intersection point and carrying a small amount of negative energy, $\kappa\lambda\Delta/(4x^-_{int})$, to null infinity. One may call it a "thunderpop".[14] The solution (25) is one of the static solutions that is asymptotically flat (with no radiation) on $\Im^+$. This means that there is no Hawking radiation after the thunderpop (26).

The mass remaining after the shockwave (26) has been emitted is $\delta M - \kappa\lambda\Delta/(4x^-_{int})$. One readily verifies that this is equal to the mass of the "remnant" (relative to C_0) $M_{rem} = \lambda(C^* - C_0)$. The fact that energy is exactly conserved, including terms of order $\hbar$, supports the self-consistency of our semi-classical theory. Notice that C^* and therefore the "remnant" mass is independent of the mass M of the infalling matter and of the constant C describing the initial static geometry. Even more surprising is the fact that the end-state solution with $\widehat{C} = C^*$ is the critical solution separating singular and non-singular static solutions described by Eq. (24). For $\widehat{C} > C^*$ the curvature of the solution (24) is bounded, while for $\widehat{C} < C^*$ the curvature diverges on a time-like curve $\widehat{\sigma} = \widehat{\sigma}_s$, for which $e^{-2\phi(\hat{\sigma}_s)} = 0$. In solution space, the solution (25) that has $\widehat{C} = C^*$ is the boundary between these two different classes of solutions.

Consider the late-time space-like hypersurface Σ shown in Fig. 1. Its right boundary ($\widehat{\sigma} \to \infty$) is i^0, while its left boundary is the curve $\widehat{\sigma} = \widehat{\sigma}_{cr}$, for which $e^{-2\phi} = 0$. For the critical solution we have $\partial_{x^+}(e^{-2\phi(\hat{\sigma}_{cr})}) = e^{-2\phi(\hat{\sigma}_{cr})} = 0$ and the curve $\widehat{\sigma} = \widehat{\sigma}_{cr}$ is the analytical continuation of the apparent horizon to the region $x^- > x^-_{int}$. We define $\epsilon \equiv \widehat{\sigma} - \widehat{\sigma}_{cr}$, and calculate the metric near $\epsilon = 0$. From (25) we get

$$ds^2 \to \frac{-d\hat{t}^2 + d\epsilon^2}{2\lambda^2\epsilon^2 + \mathcal{O}(\epsilon^3)}, \tag{27}$$

where $\widehat{t} = \frac{1}{2}(\widehat{\sigma}^+ + \widehat{\sigma}^-)$. An important feature of (27) is that there is no linear term (in ϵ) in its denominator. The first non-vanishing term is of order ϵ^2, which means that the

geometric structure near $\epsilon = 0$ is that of an *infinite throat*. Consider for example the distance along $\widehat{t} =$ constant curves. The distance to $\widehat{\sigma} = \widehat{\sigma}_{cr}$ diverges logarithmically, exactly as it does in higher-dimensional extremal black holes. The end-state space-time is geodesically complete. One may consider this solution as an "extremal 2D black hole". On $\widehat{\sigma} = \widehat{\sigma}_{cr}$ the Ricci scalar is constant, $R^{(2)} = 4\lambda^2$ and the geometry is regular.

The most natural choice of C_0 for ground-state solution is the one with $C_0 = C^*$. This solution describes a static radiationless geometry which is regular everywhere. Any solution (14) with smaller ADM mass ($C < C^*$) has a naked singularity. In the class of solutions with no naked singularities, $C = C^*$ is the one with lowest energy. This is very similar to the linear dilaton vacuum solution (LDV) in classical dilaton gravity or to Minkowski space in Einstein gravity.[§] Also if we choose $C_0 = C^*$, then the mass remaining after the thunderpop (26) is *exactly* zero. Thus the end-state solution (25) is the static ground-state. Its geometrical structure is *independent* of the initial conditions and is a semi-infinite throat extending into the strong coupling region.

In our 2D semiclassical model, one does not recover all the information of the initial state from the end-state solution. For infalling matter described by a general $(T^f_{++})_{c\ell}$ of compact support, the solution (10) will depend only on the first two moments of $(T^f_{++})_{c\ell}$, $M = \lambda \int x^+ (T^f_{++})_{c\ell} dx^+$ and $P_+ = \int (T^f_{++})_{c\ell} dx^+$.[14] The end-state solution will still be (25), but with $\Delta = \lambda^{-2} P_+$. The information encoded in this "remnant" (or more precisely, in its past null boundary $x^- = x^-_{int}$) is only about P_+ and M. Thus in our semiclassical model this end-state solution does not qualify as the "cornucopion" of Ref. [[28]]. However, the semi-infinite throat extends to a region of very strong coupling. There may be sufficient freedom in this strong coupling region to encode more information through strong quantum gravitational effects.

In this work we constructed an action in 2D dilaton gravity and showed that, with a natural boundary condition, all evaporating black holes in our model end in a unique ground-state geometry having a semi-infinite throat.

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[§] Classical solutions with ADM mass smaller than the LDV have a naked singularity, as do Schwarzschild solutions with mass smaller than zero.

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Semi-infinite Throat as the End-state Geometry of two-dimensional Black Hole Evaporation

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ABSTRACT: We study a modified two-dimensional dilaton gravity theory which is exactly solvable in the semiclassical approximation including back-reaction. The vacuum solutions of this modified theory are asymptotically flat static space-times. Infalling matter forms a black hole if its energy is above a certain threshold. The black hole singularity is initially hidden behind a timelike apparent horizon. As the black hole evaporates by emitting Hawking radiation, the singularity meets the shrinking horizon in finite retarded time to become naked. A natural boundary condition exists at the naked singularity such that for general infalling matter-configurations the evaporating black hole geometries can be matched continuously to a unique static end-state geometry. This end-state geometry is asymptotically flat at its right spatial infinity, while its left spatial infinity is a semi-infinite throat extending into the strong coupling region.

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Hawking's discovery that black holes radiate thermally [1,2,3] gave rise to a long-standing question concerning the consequences of combining quantum theory and general relativity. [4,5,6,7] Does evolution from an initial pure state take place unitarily to a final pure state or non-unitarily to a final mixed state? Intimately linked to this question is the final geometry resulting from black hole evaporation.

Here we present a specific two-dimensional (2D) dilaton gravity model in which a black hole evaporates leaving a static semi-infinite throat as the end-state or "remnant" geometry. Our model is a modification of the CGHS model.[8] We solve the semiclassical equations and get closed-form expressions for the metric and dilaton field.

The classical 2D CGHS action [8] is

$$S_{c\ell} = \frac{1}{2\pi}\int d^2x\sqrt{-g}\left[e^{-2\phi}\left(R^{(2)} + 4(\nabla\phi)^2 + 4\lambda^2\right) - \frac{1}{2}\sum_{i=1}^{N}(\nabla f_i)^2\right], \tag{1}$$

where ϕ is the dilaton field, $R^{(2)}$ is the 2D Ricci scalar, λ is a positive constant, ∇ is the covariant derivative, and the f_i are N matter (massless scalar) fields. The action (1) describes a 2D effective theory in the throat region of a 4D almost extreme magnetically charged black hole.[9,10] It may also be regarded as a 2D arena in which some of the main questions about black hole evaporation can be studied. Among the classical solutions stemming from the action (1) are vacuum solutions, static black hole solutions, and dynamical solutions describing the formation of a black hole by collapsing matter fields. For a review see Ref. [11].

To study one loop quantum corrections and back-reaction one can use the trace-anomaly for massless scalar fields in two dimensions, $\langle T^\mu_\mu\rangle = \frac{\hbar}{24}R^{(2)}$, and find the effective action S_{PL} for which $\langle T_{\mu\nu}\rangle = -\frac{2\pi}{\sqrt{-g}}\frac{\delta}{\delta g^{\mu\nu}}S_{PL}$. This is the Polyakov-Liouville action [12]

$$S_{PL} = -\frac{\hbar}{96\pi}\int d^2x\sqrt{-g(x)}\int d^2x'\sqrt{-g(x')}R^{(2)}(x)G(x,x')R^{(2)}(x'), \tag{2}$$

where $G(x,x')$ is a Green function for ∇^2. Here we take the large N limit, in which $\hbar$ goes to zero while $N\hbar$ is held fixed. In that limit the quantum corrections for the gravitational and dilaton fields are negligible, and one need take into account only the quantum corrections for the matter (scalar) fields. The one-loop effective action is then $S_{(1)} = S_{c\ell} + NS_{PL}$. There are no known analytic solutions to this one-loop effective theory, though there are some numerical ones.[13] In order to find analytic solutions including semiclassical corrections, one can modify the action as in [14,15,16]. Our approach is similar, in that we

modify the original CGHS action (1) and find analytic solutions to the modified equations including back-reaction. However, our analytic solutions yield *closed-form* expressions for the metric and dilaton field. This allows us to fully analyze the solutions.

We add to the classical action (1) a local covariant term of one-loop order,

$$S_{\text{corr}} = \frac{N\hbar}{24\pi}\int d^2x\sqrt{-g}\left((\nabla\phi)^2 - \phi R^{(2)}\right). \tag{3}$$

Now the total modified action including the one-loop Polyakov-Liouville term is

$$S_{\text{mod}} = S_{c\ell} + S_{\text{corr}} + NS_{PL}. \tag{4}$$

Using null coordinates $z^{\pm}$ and conformal gauge $g_{++} = g_{--} = 0$, $g_{+-} = -\frac{1}{2}e^{2\rho}$ ($ds^2 = -e^{2\rho}dz^+dz^-$), the action (4) can be written in the form

$$S_{\text{mod}} = \frac{1}{\pi}\int dz^+dz^-\left[2\partial_-(\phi-\rho)\partial_+\left(e^{-2\phi} - \frac{\kappa}{2}(\phi-\rho)\right) + \lambda^2 e^{2(\rho-\phi)} + \frac{1}{2}\sum_{i=1}^{N}\partial_+ f_i\partial_- f_i\right], \tag{5}$$

where $\kappa = \frac{N\hbar}{12}$. [From the point of view of string theory, the action (5) with free fields $X \equiv e^{-2\phi}$ and $Y \equiv \phi - \rho$ (which are flat target space coordinates), describes a conformal field theory with tachyon and dilaton backgrounds $\mathrm{T} = -4\lambda^2 e^{-2Y}$ and $\Phi = -2X + 2\kappa Y$.[17]] The action (5) is also invariant under the transformation† [14] $\delta\phi = \delta\rho = \epsilon e^{2\phi}$, with the conservation equation $\partial_\mu\partial^\mu(\phi-\rho) = 0$. We therefore can complete the gauge fixing by choosing the "Kruskal coordinates," $x^{\pm}(z^{\pm})$, in which $\phi(x^+,x^-) = \rho(x^+,x^-)$. In this Kruskal gauge the equations of motion derived from the modified action (5) are *exactly* the same as the classical ones

$$\partial_{x^+}\partial_{x^-}\left(e^{-2\rho(x^+,x^-)}\right) = \partial_{x^+}\partial_{x^-}\left(e^{-2\phi(x^+,x^-)}\right) = -\lambda^2 \tag{6}$$

$$\partial_{x^+}\partial_{x^-} f_i(x^+,x^-) = 0, \tag{7}$$

while the constraints get modified by non-local terms $t_{\pm}(x^{\pm})$ arising from the Polyakov-Liouville action. In conformal gauge, one can use the trace anomaly of N massless scalar fields f_i to obtain $\langle T^f_{+-}\rangle = -\kappa\partial_+\partial_-\rho$ and integrate [18,19,20] the equation $\nabla^\mu\langle T^f_{\mu\nu}\rangle = 0$ to get the quantum corrections to the energy-momentum tensor of the f_i matter fields

$$\langle T^f_{\pm\pm}\rangle = \kappa\left(\partial^2_\pm\rho - (\partial_\pm\rho)^2 - t_\pm(z^\pm)\right), \tag{8}$$

† Unlike in the RST model, [14], in this model the transformation is *exactly* the same as in the classical case.

where $t_\pm(z^\pm)$ are integration functions determined by the specific quantum state $|\Psi\rangle$ corresponding to the expectation value $\langle\Psi|T^f_{\mu\nu}|\Psi\rangle \equiv \langle T^f_{\mu\nu}\rangle$. These functions can be determined by boundary conditions. Alternatively, Eq. (8) can be obtained by varying NS_{PL}. Then the functions $t_\pm(z^\pm)$ arise from the homogeneous part of the Green function in Eq. (2). Our modified constraints (in Kruskal gauge) are

$$\frac{\delta S_{\mathrm{mod}}}{\delta g^{\pm\pm}} = 0 \;\Rightarrow\; -\partial^2_{x^\pm}\left(e^{-2\phi(x^+,x^-)}\right) - (T^f_{\pm\pm})_{c\ell} + \kappa t_\pm(x^\pm) = 0, \tag{9}$$

where $(T^f_{\pm\pm})_{c\ell} = \frac{1}{2}\sum_{i=1}^N (\partial_{x^\pm} f_i)^2$ is the classical (zero order in $\hbar$) contribution to the energy-momentum tensor of the f_i matter fields. $\langle T^f_{\mu\nu}\rangle$ in (8) is the one-loop quantum correction of order $\hbar$, so the full energy-momentum tensor of the f-fields is $(T^f_{\mu\nu})_{c\ell} + \langle T^f_{\mu\nu}\rangle + O(\hbar^2)$.

For a given classical matter distribution and a given $t_\pm(x^\pm)$ one finds the solution for the equations of motion (6) with the constraints (9):

$$\begin{aligned} e^{-2\phi} = e^{-2\rho} = -\lambda^2 x^+ x^- - \int^{x^+} dx_2^+ \int^{x_2^+} dx_1^+ \left[(T^f_{++})_{c\ell} - \kappa t_+(x_1^+)\right] \\ - \int^{x^-} dx_2^- \int^{x_2^-} dx_1^- \left[(T^f_{--})_{c\ell} - \kappa t_-(x_1^-)\right] + a_+ x^+ + a_- x^- + b \end{aligned} \tag{10}$$

where $a_\pm$ and b are constants. First, let us consider the linear dilaton flat space-time solution, $e^{-2\phi} = e^{-2\rho} = -\lambda^2 x^+ x^-$. It corresponds to the choice $(T^f_{\mu\nu})_{c\ell} = 0$ and $t_\pm(x^\pm) = a_\pm = b = 0$. To determine the corresponding quantum state $|\Psi\rangle$ one must calculate $\langle T^f_{\pm\pm}\rangle$ in (8) using the given $t_\pm(x^\pm)$. In flat coordinates $\sigma^\pm$, which are related to the Kruskal coordinates $x^\pm$ by the conformal coordinate transformation $\pm\lambda x^\pm = e^{\pm\lambda\sigma^\pm}$, the expectation values (8) are $\langle T^f_{\pm\pm}(\sigma^\pm)\rangle = \frac{\kappa\lambda^2}{4}$. We see that unlike in the RST model, in our model $\langle T^f_{\pm\pm}(\sigma^\pm)\rangle \neq 0$ for the linear dilaton solution. Because $\langle T^f_{\pm\pm}\rangle = \frac{\kappa\lambda^2}{4}$ and $\langle T^f_{+-}\rangle = 0$, the quantum state $|\Psi\rangle$ corresponding to the linear dilaton solution may describe a system in thermal equilibrium at temperature $T = \frac{\lambda}{2\pi}$.

In our model we also have *static* black hole solutions.[21] These correspond in Eq. (10) to the choice $(T^f_{\mu\nu})_{c\ell} = t_\pm(x^\pm) = a_\pm = 0$ and $b = M/\lambda$. For these solutions at future and past null infinity ($\Im^+$ and $\Im^-$, respectively) one has $\langle T^f_{\pm\pm}\rangle = \frac{\kappa\lambda^2}{4}$; the solutions evidently describe a black hole in thermal equilibrium at temperature[‡] $T_{bh} = \frac{\lambda}{2\pi}$. This is as we would

[‡] Since in 2D the Hawking temperature is mass independent, one may regard the linear dilaton solution as the zero mass limit of the static black hole solutions. This may explain the non-zero temperature of the linear dilaton solution in our model.

expect: A static black hole solution in a self-consistent semiclassical theory of Hawking radiation including back-reaction is possible only if the black hole is in thermal equilibrium with incoming radiation.

In order to find the solution corresponding asymptotically to the Minkowski vacuum we can use (8) to find the solution for which $\langle T^f_{\pm\pm}(\sigma^\pm)\rangle = 0$. The functions $t_\pm(x^\pm)$ are determined by imposing appropriate boundary conditions on $\Im^\pm$. We assume that on these boundaries the metric is flat, such that $\rho(\sigma^\pm)$ and its derivatives vanish in the asymptotically flat coordinates $\sigma^\pm$. Then the first two terms on the right-hand-side of (8) vanish on the boundary and we get

$$\langle T^f_{\pm\pm}(\sigma^\pm)\rangle|_{\text{boundary}} = -\kappa t_\pm(\sigma^\pm), \tag{11}$$

We see from (11) that the Minkowski vacuum corresponds to $t_\pm(\sigma^\pm) = 0$. To find the corresponding $t_\pm(x^\pm)$ in "Kruskal coordinates," one can use the tensor transformation of $\langle T^f_{\pm\pm}\rangle$ in Eq. (8) (under a conformal coordinate transformation) and get

$$t_\pm(x^\pm) = \left(\frac{\partial\sigma^\pm}{\partial x^\pm}\right)^2 \left(t_\pm(\sigma^\pm) - \frac{1}{2}D^S_{\sigma^\pm}[x^\pm]\right) = \frac{1}{(2x^\pm)^2}, \tag{12}$$

where $D^S_y[z]$ is the Schwarz operator $D^S_y[z] = \partial^3_y z/(\partial_y z) - \frac{3}{2}\left(\partial^2_y z/\partial_y z\right)^2$ and we use $t_\pm(\sigma^\pm) = 0$. Using (10), (12) and $(T^f_{\mu\nu})_{c\ell} = 0$, we find that the general asymptotically Minkowski vacuum solution is

$$e^{-2\phi} = e^{-2\rho} = -\lambda^2 x^+ x^- - \frac{\kappa}{4}\log(-\lambda^2 x^+ x^-) + C, \tag{13}$$

where C is a constant. In asymptotically flat coordinates $\sigma^\pm = t \pm \sigma$, we have

$$\begin{aligned} ds^2 &= \left(1 - e^{-2\lambda\sigma}(\kappa\lambda\sigma/2 - C)\right)^{-1}(-dt^2 + d\sigma^2) \\ \phi(\sigma) &= -\lambda\sigma + \log\left(1 - e^{-2\lambda\sigma}(\frac{\kappa\lambda}{2}\sigma - C)\right). \end{aligned} \tag{14}$$

This solution is static, depending on the spatial coordinate σ alone. On the boundaries $\Im^\pm$, the solution approaches the linear dilaton flat space-time solution, justifying our assumption. The reason this solution with no radiation at $\Im^\pm$ and the earlier ones with radiation there all asymptotically approach the linear dilaton flat space-time solution is that the coupling, $e^{2\phi}$, of the matter to the geometry vanishes exponentially fast at $\Im^\pm$.

Before we turn to the question of the ground-state solution, let us consider the ADM masses of the various solutions we have found. Suppose that we can choose as our ground-state one of the radiationless solutions (14) with $C = C_0$, where C_0 is a constant yet to be determined. Then the ADM mass [22,23] of any other static solution (14) is $\lambda(C - C_0)$. On the other hand, the ADM mass of the linear dilaton solution as well as the static black hole solutions (relative to this ground-state) is infinite. This is already clear from the fact that these solutions have non-vanishing radiation on $\Im^{\pm}$ and can be checked explicitly by using the ADM mass definition.[22] These considerations make it plausible that one of the static solutions (14) should be the ground-state. We will see later that there exists a natural lower limit on C_0 which gives the preferred ground-state of lowest ADM mass.

We next turn to the dynamical scenario in which the space-time is initially described by one of the static solutions in (14) (not necessarily the ground state solution C_0), and in which a black hole is formed by collapsing matter fields. First we consider the simple shock wave solution, but our results can be easily extended to general infalling matter configurations. The shock wave of infalling matter is described by $(T^f_{++})_{c\ell} = \frac{M}{\lambda x_0^+}\delta(x^+ - x_0^+)$ and $(T^f_{--})_{c\ell} = 0$.[8] Unlike in the RST model, here we have a general initial static geometry, and the shock wave forms a black hole only if M, the energy of the shock wave, is above a certain threshold energy. We assume that M is above that threshold. Integrating $(T^f_{++})_{c\ell}$ in (10) and using (12) and $a_\pm = 0$, we find the evaporating black hole solution

$$e^{-2\phi} = e^{-2\rho} = -\lambda^2 x^+ x^- - \frac{\kappa}{4}\log(-\lambda^2 x^+ x^-) - \frac{M}{\lambda x_0^+}(x^+ - x_0^+)\Theta(x^+ - x_0^+) + C, \tag{15}$$

where $\Theta(x)$ is the standard step function.

Before the shock wave, i.e., in the region $x^+ < x_0^+$, we have a static solution (14) which is not globally flat. If $\frac{\kappa}{4}[1 - \log(\kappa/4)] + C < 0$, then the scalar curvature *diverges* on a timelike curve $\sigma = \sigma_s$, for which $e^{-2\phi(\sigma_s)} = 0$. Of course this is a region of strong coupling, and one would expect to have higher order quantum corrections there. On the other hand, if $\frac{\kappa}{4}[1-\log(\kappa/4)]+C > 0$, the scalar curvature is bounded. Then the region $\{x^+ \geq 0, x^- \leq 0\}$ is geodesically incomplete and one can analytically extend it to $x^- > 0$ and $x^+ < 0$. Also in this case there is a region of strong coupling near $\sigma = \sigma_{min} = -\frac{1}{2\lambda}\log(\frac{\kappa}{4})$. In the semiclassical approximation, one avoids the strong coupling region by imposing boundary conditions on a suitable time-like hypersurface.[24,14,25] For the static solutions (14), $\langle T^f_{++}(\sigma^\pm)\rangle$ and $\langle T^f_{--}(\sigma^\pm)\rangle$ are constant on *any* time-like hypersurface $\sigma =$ const. Moreover

$$\langle T^f_{++}(\sigma^\pm)\rangle\big|_{\sigma=\sigma_0} = \langle T^f_{--}(\sigma^\pm)\rangle\big|_{\sigma=\sigma_0} \qquad \text{for any constant } \sigma_0. \tag{16}$$

This means that we can limit our model to a region in which the semiclassical approximation is valid by imposing reflecting boundary conditions (16) on any time-like hypersurface $\sigma = \sigma_0$ that lies outside the region of strong coupling (these boundary conditions are also conformal [25]). The geometry before the shock wave is therefore a static geometry, defined in the region $\sigma > \sigma_s$ in the case $\frac{\kappa}{4}[1-\log(\kappa/4)] + C < 0$ (or $\sigma > \sigma_{min}$ in the case $\frac{\kappa}{4}[1-\log(\kappa/4)] + C > 0$), with reflecting boundary conditions on $\sigma = \sigma_s + \delta$ (or on $\sigma = \sigma_{min} + \delta$) where δ is an arbitrary small positive constant.

The solution to the future of the shock wave ($x^+ > x_0^+$) is (see (15))

$$e^{-2\phi} = e^{-2\rho} = -\lambda^2 x^+(x^- + \Delta) - \frac{\kappa}{4}\log(-\lambda^2 x^+ x^-) + \frac{M}{\lambda} + C, \tag{17}$$

where $\Delta = \frac{M}{\lambda^3 x_0^+}$. This solution is asymptotically flat and describes a black hole with a singularity at $e^{-2\phi} = 0$. The black hole singularity curve is

$$-\lambda^2 x_s^+(x_s^- + \Delta) - \frac{\kappa}{4}\log(-\lambda^2 x_s^+ x_s^-) + \frac{M}{\lambda} + C = 0. \tag{18}$$

Initially the singularity is behind an apparent horizon $\partial_+ e^{-2\phi} = 0$,[26,[10]] which is the curve

$$-\lambda^2 x_h^+(x_h^- + \Delta) = \frac{\kappa}{4}. \tag{19}$$

When the apparent horizon is formed, the black hole starts radiating. One can see this by calculating $\langle T^f_{\mu\nu}\rangle$ at future null infinity ($x^+ \to \infty$). From (17) we see that the asymptotically flat coordinates on $\Im^+$ are $\widehat{\sigma}^\pm$, related to $x^\pm$ by the conformal coordinate transformation, $\lambda\widehat{\sigma}^+ = \log(\lambda x^+)$ and $-\lambda\widehat{\sigma}^- = \log(-\lambda(x^- + \Delta))$. Using (11) and (12) we get

$$\langle T^f_{--}(\widehat{\sigma}^\pm)\rangle|_{\Im^+} = \frac{\kappa\lambda^2}{4}\left(1 - \frac{1}{(1+\lambda\Delta e^{\lambda\hat{\sigma}^-})^2}\right). \tag{20}$$

This is the "standard" Hawking radiation in 2D, where the Hawking temperature $T_H = \frac{\lambda}{2\pi}$ is a constant.[8] One can further verify that when the black hole evaporates over a long period of time, i.e., if $M >> \kappa\lambda$, the spectrum of the Hawking radiation is indeed Planckian.[[2],27]

As the black hole evaporates by emitting Hawking radiation, the apparent horizon shrinks and eventually meets the singularity in a *finite* proper time. They intersect at (see Fig. 1)

$$x^+_{int} = \frac{1}{\lambda^2\Delta}\left(e^{(\frac{4(M+\lambda C)}{\kappa\lambda}+1)} - \frac{\kappa}{4}\right) \qquad \text{and} \qquad x^-_{int} = -\Delta\Big(1 - \frac{\kappa}{4}e^{-(\frac{4(M+\lambda C)}{\kappa\lambda}+1)}\Big)^{-1}. \tag{21}$$

At this point the singularity becomes naked. We show that it is possible to impose a boundary condition in which a weak shock wave emanates from the intersection point, resulting in a solution that is stable (having non-negative ADM mass), conserves energy, and is continuous with the metric defined to the past of the null hypersurface $x^- = x^-_{int}$.

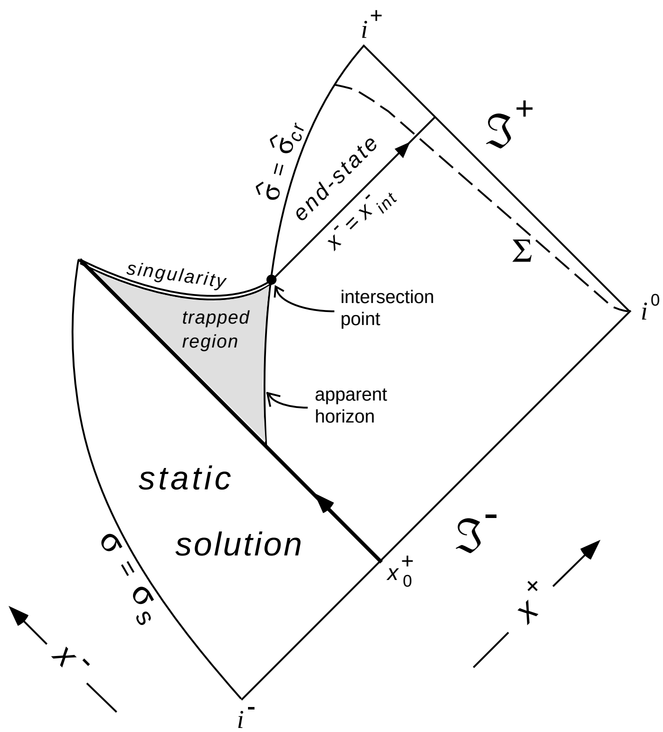

Fig. 1: Penrose diagram describing formation and subsequent evaporation of a black hole in our model.

Before considering the solution to the future of the null hypersurface $x^- = x^-_{int}$ (the end-state solution), we calculate the total amount E_{rad} of energy radiated during the evaporation. Integrating (20) over $\Im^+$ (up to x^-_{int}) gives

$$E_{rad} = \int_{-\infty}^{\hat{\sigma}^-_{int}} \langle T^f_{--}(\hat{\sigma}^-)\rangle d\hat{\sigma}^- = M + \lambda C - \frac{\kappa\lambda}{4}\left(\log(\kappa/4) - 1\right) - \frac{\kappa\lambda\Delta}{4x^-_{int}}, \qquad (22)$$

where $\hat{\sigma}^-_{int} = \hat{\sigma}^-(x^-_{int})$. The result (22) is *exact*. The ADM mass [22] of the dynamical solution (15) (relative to the ground state $C = C_0$) is $M_{ADM} = M + \lambda(C - C_0)$. We

see that the black hole radiates almost all of its initial energy. The unradiated mass δM remaining as $x^- \to x^-_{int}$ (which is the Bondi mass) is

$$\delta M = M_{ADM} - E_{rad} = \frac{\kappa\lambda}{4}\left(\log(\kappa/4) - 1\right) - \lambda C_0 + \frac{\kappa\lambda\Delta}{4x^-_{int}}. \tag{23}$$

We now consider the solution to the future of the point of intersection (x^+_{int}, x^-_{int}). A natural candidate for such an end-state in our model is one of the static solutions (14), so we try to find boundary conditions such that the solution (17) is continuously matched to one of the static solutions (14). Remember that the asymptotically flat coordinates are $\widehat{\sigma}^{\pm}$, so one should replace σ in (14) with $\widehat{\sigma} = \frac{1}{2}(\widehat{\sigma}^+ - \widehat{\sigma}^-)$. In the $x^{\pm}$ coordinates the corresponding static solution is (see (13))

$$e^{-2\phi} = e^{-2\rho} = -\lambda^2 x^+(x^- + \Delta) - \frac{\kappa}{4}\log(-\lambda^2 x^+(x^- + \Delta)) + \widehat{C}. \tag{24}$$

We would like to see if there exists a constant $\widehat{C} = C^*$, such that on the null hypersurface $x^- = x^-_{int}$ the solutions (17) and (24) can be matched continuously. This is indeed the case and from (21), (17) and (24) we get $C^* = -\frac{\kappa}{4}(1 - \log(\kappa/4))$. The end-state solution, or "remnant", is therefore

$$e^{-2\phi} = e^{-2\rho} = -\lambda^2 x^+(x^- + \Delta) - \frac{\kappa}{4}\log(-\lambda^2 x^+(x^- + \Delta)) - \frac{\kappa}{4}(1 - \log(\kappa/4)), \tag{25}$$

where $x^- > x^-_{int}$. From the constraint equations (9) we find that

$$(T^f_{--}(\widehat{\sigma}^-))_{c\ell} = \frac{1}{2}\sum_{i=1}^{N}(\partial_- f_i)^2 = \frac{\kappa\lambda\Delta}{4x^-_{int}}\delta(\widehat{\sigma}^- - \widehat{\sigma}^-_{int}). \tag{26}$$

This describes a shock wave originating at the intersection point and carrying a small amount of negative energy, $\kappa\lambda\Delta/(4x^-_{int})$, to null infinity. One may call it a "thunderpop".[14] The solution (25) is one of the static solutions that is asymptotically flat (with no radiation) on $\Im^+$. This means that there is no Hawking radiation after the thunderpop (26).

The mass remaining after the shockwave (26) has been emitted is $\delta M - \kappa\lambda\Delta/(4x^-_{int})$. One readily verifies that this is equal to the mass of the "remnant" (relative to C_0) $M_{rem} = \lambda(C^* - C_0)$. The fact that energy is exactly conserved, including terms of order $\hbar$, supports the self-consistency of our semi-classical theory. Notice that C^* and therefore the "remnant" mass is independent of the mass M of the infalling matter and of the constant C describing the initial static geometry. Even more surprising is the fact that the

end-state solution with $\widehat{C} = C^*$ is the critical solution separating singular and non-singular static solutions described by Eq. (24). For $\widehat{C} > C^*$ the curvature of the solution (24) is bounded, while for $\widehat{C} < C^*$ the curvature diverges on a time-like curve $\widehat{\sigma} = \widehat{\sigma}_s$, for which $e^{-2\phi(\hat{\sigma}_s)} = 0$. In solution space, the solution (25) that has $\widehat{C} = C^*$ is the boundary between these two different classes of solutions.

Consider the late-time space-like hypersurface Σ shown in Fig. 1. Its right boundary $(\widehat{\sigma} \to \infty)$ is i^0, while its left boundary is the curve $\widehat{\sigma} = \widehat{\sigma}_{cr}$, for which $e^{-2\phi} = 0$. For the critical solution we have $\partial_{x^+}(e^{-2\phi(\hat{\sigma}_{cr})}) = e^{-2\phi(\hat{\sigma}_{cr})} = 0$ and the curve $\widehat{\sigma} = \widehat{\sigma}_{cr}$ is the analytical continuation of the apparent horizon to the region $x^- > x^-_{int}$. We define $\epsilon \equiv \widehat{\sigma} - \widehat{\sigma}_{cr}$, and calculate the metric near $\epsilon = 0$. From (25) we get

$$ds^2 \to \frac{-d\hat{t}^2 + d\epsilon^2}{2\lambda^2\epsilon^2 + \mathcal{O}(\epsilon^3)}, \tag{27}$$

where $\widehat{t} = \frac{1}{2}(\widehat{\sigma}^+ + \widehat{\sigma}^-)$. An important feature of (27) is that there is no linear term (in ϵ) in its denominator. The first non-vanishing term is of order ϵ^2, which means that the geometric structure near $\epsilon = 0$ is that of an *infinite throat*. Consider for example the distance along $\widehat{t} =$ constant curves. The distance to $\widehat{\sigma} = \widehat{\sigma}_{cr}$ diverges logarithmically, exactly as it does in higher-dimensional extremal black holes. The end-state space-time is geodesically complete. One may consider this solution as an "extremal 2D black hole". On $\widehat{\sigma} = \widehat{\sigma}_{cr}$ the Ricci scalar is constant, $R^{(2)} = 4\lambda^2$ and the geometry is regular.

The most natural choice of C_0 for ground-state solution is the one with $C_0 = C^*$. This solution describes a static radiationless geometry which is regular everywhere. Any solution (14) with smaller ADM mass ($C < C^*$) has a naked singularity. In the class of solutions with no naked singularities, $C = C^*$ is the one with lowest energy. This is very similar to the linear dilaton vacuum solution (LDV) in classical dilaton gravity or to Minkowski space in Einstein gravity.[§] Also if we choose $C_0 = C^*$, then the mass remaining after the thunderpop (26) is *exactly* zero. Thus the end-state solution (25) is the static ground-state. Its geometrical structure is *independent* of the initial conditions and is a semi-infinite throat extending into the strong coupling region.

In our 2D semiclassical model, one does not recover all the information of the initial state from the end-state solution. For infalling matter described by a general $(T^f_{++})_{c\ell}$ of

[§] Classical solutions with ADM mass smaller than the LDV have a naked singularity, as do Schwarzschild solutions with mass smaller than zero.

compact support, the solution (10) will depend only on the first two moments of $(T^f_{++})_{c\ell}$, $M = \lambda \int x^+ (T^f_{++})_{c\ell} dx^+$ and $P_+ = \int (T^f_{++})_{c\ell} dx^+$.[14] The end-state solution will still be (25), but with $\Delta = \lambda^{-2} P_+$. The information encoded in this "remnant" (or more precisely, in its past null boundary $x^- = x^-_{int}$) is only about P_+ and M. Thus in our semiclassical model this end-state solution does not qualify as the "cornucopion" of Ref. [28]. However, the semi-infinite throat extends to a region of very strong coupling. There may be sufficient freedom in this strong coupling region to encode more information through strong quantum gravitational effects.

In this work we constructed an action in 2D dilaton gravity and showed that, with a natural boundary condition, all evaporating black holes in our model end in a unique ground-state geometry having a semi-infinite throat.

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